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SUPERPOPULATION MODELS OF LABOR SUPPLY AND CONSUMER EXPENDITURES

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The major objective of this research was development of a new class of models suitable for more extensive study of worker and consumer behavior. It was the intention that such models would account for individual preferences as well as labor market conditions. That foundation is laid by development of superpopulation models, ~~as reported herein~~ Additional objectures included using actual survey data to estimate the models. In this

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report are reported estimates of over twenty models of different populations. Results, although preliminary and subject to modification, indicate that further development of these models and techniques will prove worthwhile and beneficial to manpower planning and policy making and many allied areas of economic research.

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I. INTRODUCTION

Advent of the all volunteer force brings the necessity for hiring military manpower directly from the private labor market. Military recruiting must now compete with private and public firms for a limited supply of manpower. Successful recruiting and retention of the all volunteer force is enhanced by increased understanding of the decision process followed by various sectors of the labor force. The main purpose of this project is to lay foundation for new studies in consumer and worker behavior. Ultimately this foundation should lead to greater understanding of labor markets and, consequently, to more successful recruiting and retention tactics.

The major objective of this research was development of a new class of models suitable for more extensive study of worker and consumer behavior. It was the intention that such models would account for individual preferences as well as labor market conditions. That foundation is laid by development of superpopulation models as reported herein. Additional objectives included using actual survey data to estimate the models. In this paper are reported estimates of over twenty models of different populations. Therefore, it is concluded that the superpopulation model research described in this report successfully meets the research objectives of this project.

As with any exploratory research, the results discussed in this report are preliminary and subject to modification. There remains extensive analysis and interpretation of these results and much further development of superpopulation models. However, the work done clearly indicates that further development of these models and techniques will prove worthwhile and

beneficial not only in manpower planning and policy-making, but in many allied areas of economic research.

The outline of the remainder of this report is as follows. In chapter II the basic concept of a superpopulation model is introduced and explained along with examples of superpopulation models. The model developed for this project is presented in some detail in chapter III. Chapter IV presents the estimation methods developed for this project. Data used for estimating the model are discussed in chapter V. Finally, chapter VI contains estimates of superpopulation models for several occupation classifications and a brief discussion of the results.

II. SUPERPOPULATION CONCEPT FOR STUDYING LABOR SUPPLY BEHAVIOR

A. Introduction

The goal of this study is to create methods for successfully explaining consumer and worker behavior as observed and recorded in sample surveys. The particular methods chosen reflect the fact that most data measuring consumer expenditures and supply of factor time are represented in sample surveys. Such surveys are generally conducted under aegis of governmental agencies. They are almost never based on simple random samples. They are frequently used for drawing inferences regarding economic behavior in general circumstances rather than merely the economic behavior of the particular population from which the sample was drawn. These considerations have entered into the methods developed and applied in this research project.

One baseline objective of this study is to provide models of populations of consumers and workers from which sample surveys may be drawn. Models are designed to characterize the populations through modeling the distributions associated with survey populations. It is recognized that models of populations may succeed in characterizing and explaining observed behavior where models based only on sample moments may fail. In any event, models of populations will always contain more information regarding population behavior than models of sample moments.

A second baseline objective of this study is to provide model components that explain and characterize aggregate, or per capita, responses to dynamic economic conditions. Consumer and worker responses to price inflation, unemployment, changes in real wages, and changes in real incomes need to be explained if adequate explanations of consumer behavior are to be given. Models providing adequate explanations of observed behavior may allow for

prediction of changing patterns to be expected as responses to changing economic conditions. Not only do we wish to explain observed patterns, but we wish to anticipate conditions that will lead to new patterns. By using such predictions it may be possible to forecast economic observations based on other survey samples. This kind of process allows for management of control variables in such a way that individual and organizational goals are met. One goal of this study is to develop a modeling system that will allow for explaining observed patterns of economic behavior. The modeling system should allow creation of models that allow prediction of changes in patterns and evolution of new patterns observed as consumers respond to dynamic economic conditions affecting their spending and earning decisions.

Meeting these goals will lay foundation for study of consumer responses to stimuli such as wages, benefits, and working conditions. However, it is recognized that a modeling system will need to go beyond the usual models of sample moments. It will need to model population characteristics in a way that allows explanation and prediction of patterns of economic behavior. While this goal cannot be entirely met in a study of small compass, such as this one, considerable progress has been made nevertheless. The superpopulation technique offers a modeling system of the requisite qualities.

B. Concept of a Superpopulation

Most econometric studies, in labor economics and elsewhere, proceed according to rules of inference and estimation formed during the times of R. A. Fisher. These rules are based on the assumption that sample point (scalars or vectors) were drawn as independent and identically distributed random variables from a single hypothetical population. They also assume

that the sample points drawn form a simple random sample. The techniques of estimation and inference derived from these assumptions are known as "Fisherian Inference." Application of Fisherian problems gives useful and elegant results if the basic assumptions about the sample are accurate. However, the optimal properties of estimators and inference techniques may disappear if these assumptions fail to hold.

Since survey data are the primary source of economic observations, and are likely to continue so, it is important to recognize that survey data frequently are not collected as simple random samples. Thus, it is also important to investigate the consequences of using a Fisherian approach to estimation and inference when its underlying assumptions do not hold. Such investigations have led to creation of the Fixed Population methods of inference and estimation.

The Fixed Population approach is based on the fact that samples are drawn from finite populations with fixed collections of members at the time the sample is chosen. For example, sample units for the 1972-73 Consumer Expenditure Survey were drawn from a finite population consisting of addresses from the 1970 Census of Population plus new addresses taken from housing construction reports. Thus, the fixed population consisted of persons residing at that finite collection of addresses. Sample units were chosen from that fixed population according to a sampling scheme. The sampling scheme definitely did not lead to a simple random sample. Likewise, the sample observations in the National Longitudinal Survey were drawn from a fixed population according to a sampling scheme that is not simple random sampling. When sampling from a fixed population, such as with the CES and NLS studies, we say that we are using a Fixed Population Model.

The Fixed Population Model framework differs markedly from that of the Fisherian Model. Some differences lie in the formulas for estimating functions of parameters, while desirable properties of the estimators (unbiasedness, minimum variance, efficiency and sufficiency) remain the same. Other differences lie in the alternative definitions of parameters. In the Fisherian framework we postulate a distribution function $F(X;\theta)$ for the population, where θ is a parameter vector defined on some domain: $\theta\varepsilon\Omega$. In contrast, the parameters for the fixed population are $(y_1, \ldots, y_N)$, its elements, also defined on a domain: $(y_1, \ldots, y_N)\varepsilon D$. These different populations naturally lead to different techniques of parameter estimation and statistical inference. In one case we focus on functions of θ and in the other on functions of $(y_1, \ldots, y_N)$.

The finite population and infinite population approaches use the same concepts and criteria for inference and estimation. They differ in the estimator and inference formulas because each has specialized definitions of "parameter," "sample design," "population," etc.

These two approaches to sampling, estimation and inference should not be thought of as alternatives competing for use. Rather, each is applicable when its basic assumptions hold. The Fisherian approach is appropriate when the sample is from an hypothetically infinite population and is a simple random sample. Such conditions arise frequently in engineering applications, such as quality control, and in natural sciences, such as atomic physics and genetics. In these cases the basic propositions of simple random sampling from an hypothetically infinite population hold. On the other hand, in social science applications, the finite population approach is more often appropriate. For example, unemployment rates apply to particular finite

populations of workers at specific dates, as do other socio-economic characteristics. In these cases the finite population approach is clearly the appropriate one.

There is a third case that requires an approach different from both the Fisherian and the finite population approaches. Frequently, interest centers on characteristics of a finite population that are common to many finite populations like the existing one. For example, it may be of interest to know some labor force behavior characteristics of a given population at a specific time, but it may also be of interest to know some characteristics of a class of possible populations in identical or similar circumstances. The same may be said of consumer behavior characteristics. Studying the behavior of a certain finite population as representative of a class of finite populations may be worthwhile. Such studies may be thought of as occurring in two steps: first, the finite population of size N is selected from a conceptually infinite population; second, a sample of size $n \leq N$ is selected according to some sampling scheme from the finite population. The sample is used to study both the characteristics of the finite population as represented by the parameter vector $(y_1, \ldots, y_N)$ and the characteristics of the conceptually infinite population as represented by a distribution function $G(y;\omega)$, $\omega \varepsilon \Xi$. This third approach is called the Superpopulation approach to survey sampling. There are several interpretations of survey sampling that lead to the superpopulation concept. Some of them are reviewed in Cassel, Saerndal, and Wretman (CSW hereafter) (1977, pp. 2-3, 80-82). The superpopulation approach is an alternative to the Fisher and finite population approaches. Decision to use the superpopulation approach is based on whether its underlying assumptions hold.

It may help to clarify the concepts of superpopulations, finite populations and survey sampling if we consider an example. In Standard Mathematical Tables, 18th Edition, (Selby, ed., 1970), there is a "Table of Random Units, 14,000" on pp. 621-625. These random units constitute a finite population of size N = 14,000. They can be thought of as the values of uniform random variables drawn from the infinite population made up of all possible uniform random variable values appearing at a noncountable set of addresses. This particular finite population was created by selecting 14,000 of those addresses and recording the associated variable results. It is often convenient to draw smaller subsamples from this finite population. Such samples are ordinarily used for other purposes independent of their origin. However, they could be used to estimate parameters from the finite population or from the superpopulation from which the finite population was drawn. Now suppose that the finite sample of size n is drawn without knowledge of the characteristics of either the finite population or the infinite population and, in fact, that the sample was drawn in order to study them. Also recognize that the sampling schemes for drawing the finite population and the finite sample may or may not produce simple random samples and that the measurement technique may or may not provide precise measurements of the variables. (In this case, for example, we are required to truncate or round numbers in the generation process.) The process just described is similar to the process of collecting survey sample measurements on socio-economic variables in finite populations. Using this technique it is possible to study the characteristics of both the finite population and of the superpopulation.

These three alternative methods of statistical inference are summarized in TABLE II-1. They are classified according to the inference issues to be

TABLE II-1

Target Parameters or Characteristics	Sample Procedure		
	Repeated sampling from a fixed finite population	Repeated simple random sampling from a fixed infinite population	Repeated two-step sampling from an infinite population
Parameters of finite population	Classical finite population sampling theory = Case 4	Not feasible	Superpopulation theory for finite population sampling = Case 1
Parameters of infinite superpopulation	Not feasible	Classical Fisherian inference = Case 5	Inference on infinite population parameters from two-step sampling procedure = Case 2
Characteristics of infinite population	Not feasible	Classical nonparametric inference = Case 6	Non-parametric inference on infinite population from two-step sampling procedure = Case 3

studied. This report is primarily concerned with Case 2. Cases 1 and 3 will be treated in other papers. Reflection on empirical labor supply studies and on TABLE II-1 shows that many studies have actually mixed samples from Case 4 or Case 1 with inference procedures from Case 5. This is true even when sampling weights are used in calculations. The result is that reliable interpretation of these studies is not possible at the present time. This report contains a suggestion for using the superpopulation model approach to studying survey data.

Superpopulation modeling consists of suggesting distribution functions that characterize the infinite population of the two-step sampling procedure. For Case 1 this often reduces to studies of the basic linear regression model. This case has developed rapidly over the last few years. The book by CSW (1977) gives a survey of results through 1976. Results since 1976 can be found by checking the Current Index to Statistics, annual volumes, published by ASA/IMS. The research reported here considers extensions of superpopulation modeling to Cases 2 and 3, with special reference to labor supply behavior and theories.

The basic step in superpopulation modeling for Cases 2 and 3 is introduction of an hypothetical distribution function to characterize the superpopulation. The form of this distribution may be known by construction, as with Monte Carlo experiments, or it may be deduced from experimental conditions and design, as with measurement of radioactive emissions. On the other hand, the form of the distribution may be unknown, as in the case of socio-economic data. In such cases proposition of distribution functions for the superpopulation and testing such propositions becomes an essential part of the modeling process. Basic procedures of superpopulation modeling are illustrated as superpopulation models for study of labor supply behavior and theories are proposed.

C. Superpopulation Models

Empirical labor supply studies are usually based on either aggregate or individual data from sample surveys. The sample surveys are drawn from finite populations, usually using sampling designs that are not simple random sample designs. Stated purposes of the studies indicate that what is sought

are answers to questions about behavior in all populations similar to the one sampled. These conditions for sampling and inference naturally indicate the superpopulation approach of Case 2 as the appropriate one for labor supply studies.

It is recognized that classical theories of consumer behavior, including theories about consumption expenditure, investment, and labor supply, usually regard consumers' allocations of time and money as two-step processes. The first step in each case is determination of spendable income through labor supply decisions. The second step is allocation of spendable income to expenditures on various commodities and investment. It is not possible at the present time to reject or accept the hypothesis of separability. A good beginning toward testing this hypothesis has been made by Barnett (1979) [see also Barnett's references to similar work], but there remains much work before a tentative conclusion is adopted. In this project the separability of consumption and labor supply activities is not assumed. However, the superpopulation approach used here does permit considerable simplification through the use of marginal and conditional distribution functions.

Superpopulation Models for Consumer and Worker Behavior

For a definite population of consumers the joint probability density function

(2.1) $f^*(M_1, \ldots, M_n, Z, L, S)$

of individual consumers' planned expenditures on single commodities

$M_i, i = 1, 2, \ldots, n;$

planned total expenditure on all commodities and securities, Z; planned earnings from sale of factor time, L; and planned income from asset earnings,

transfer payments, and borrowings, S; is everywhere continuous and has its magnitudes confined to the domain

$$
(2.2)\quad
\begin{aligned}
&M_i \geqq \lambda_i,\ \lambda_i \geqq 0,\ i = 1, 2, \ldots, n.\\
&Z \geqq \lambda,\\
&\lambda = \sum_{i=1}^{n} \lambda_i;\\
&L \geqq 0\\
&S \geqq 0.
\end{aligned}
$$

For each transaction period λ_i is constant. We do not assume, however, that any of the λ_i necessarily remains constant from transaction period to transaction period. Indeed, we do assume that λ_i depends on the average transaction price, P_i, where

(2.3) $\lambda_i = P_i \gamma_i,\ i = 1, 2, \ldots, n,$

where γ_i itself is a function of population characteristics (other than) P_i, $i = 1, 2, \ldots, n$, that may undergo change from period-to-period.

In some of the alternative superpopulation models we shall consider in this article, we assume that there is a finite upper bound, Y, for individual consumers' planned total expenditures on commodities and securities, Z. For such a SPM, the non-zero magnitudes of the joint density function f* ($M_1, \ldots, M_n$, Z, L, S) are confined to the domain

$$
(2.4)\quad
\begin{aligned}
&\lambda_i \leqq M_i < Z - (\lambda - \lambda_i)\\
&i = 1, 2, \ldots, n,\\
&\lambda \leqq Z < Y.
\end{aligned}
$$

For a given transaction period the parameter Y is constant. We do not assume,

however, that Y remains constant from period to period. On the contrary, we do expect Y to change from period to period in response to changes in population characteristics such as (but not limited to) average transaction wage-rates, average transaction period interest rates, current and lagged.

It will be very convenient to the reader if we express the density function $f^* (M_1, \ldots, M_n, Z, L, S)$ in terms of translated variables and parameters. The translated variables m_i, z, L, s, are defined by

$$M_i = m_i + \lambda_i, \quad i = 1, 2, \ldots, n$$

(2.5)

$$Z = z + \lambda,$$

$$S = s.$$

Also $\quad x = z + L + s, \quad X = x + \lambda$, and

(2.6) $\quad Y = y + \lambda$

Under the change of variables (2.5) and (2.6) we derive the following joint density function of the translated variables:

$$f (m_1, \ldots, m_n, z, L, s)$$

(2.7)

$$= f^* [m_1 + \lambda_1, \ldots, m_n + \lambda_n, z + \lambda, L, s]$$

$$m_i \geqq 0, \; i = 1, 2, \ldots, n;$$

$$m \leqq z < y; \; m = \sum_{i=1}^{n} m_i,$$

$$L \geqq 0,$$

$$s \geqq 0;$$

$$= 0 \text{ otherwise.}$$

The present report is primarily concerned with aspects of the joint marginal density function of planned aggregate expenditures on commodities, m, total expenditures on commodities and securities, z, planned earnings from the sale of factor time, L, and other income, s, which is defined by

$$h(m, z, L, s) = \int f(m_1, \ldots, m_n, z, L, s)\, dm_1 \ldots dm_n \tag{2.8}$$

$$m_1 + \ldots + m_n = m$$

The center of interest is consumers' allocations of factor time between (a) satisfying wants indirectly (by rendering paid service to the production sector) and (b) by direct self-service, i.e., leisure.

Most of the remainder of this chapter is devoted to description of a superpopulation model theory of labor supply behavior. Major components of such a theory are the marginal and conditional distribution functions derivable from (2.8). Marginal and conditional distributions form a natural division of the subject matter. Marginal distributions of m, z, L, and s are the primary empirical subject matter for explanation in this theory of labor supply. They constitute one major area of study. The second major area of study is composed of the conditional distributions and conditional expectations that are ancillary to the main subject. They are, nevertheless, important in several respects, including the study of aggregate labor supply behavior. The second major area includes the "Little Theory of Factor-time Supply."

Marginal Distribution Functions

Primary empirical data that SPM models are to explain are the empirical distributions of planned total expenditures on commodities, planned total

expenditures on commodities and securities, planned income from the supply of factor time to the production sector, and planned income from other sources. Consequently, the major theoretical subject matter of the SPM approach is composed of the theoretical marginal distribution functions, their characteristics and the relations among them.

The beginning point in all of this work is the joint marginal distribution function (hereafter d.f.) of m, z, L, and s: F(m, z, L, s). In this report it is assumed that F(m, z, L, s) is a member of the class of everywhere continuous distribution functions, so that it defines a joint frequency function:

$$(2.9) \qquad f(m, z, L, s) = d^4F(m, z, L, s)/dmdzdLds.$$

By integration, the various marginal joint frequency functions (hereafter f.f.) are defined: f(m, z, L), . . ., f(m, z,), . . ., f(m), f(z), f(L), f(s).

Little Theory of Planned Factor-time Supply

Nearly every model of aggregate economic activity extant includes a component for aggregate supply of labor. While there are many problems with the current methods of including the labor supply component that could be dealt with here, it is beyond the scope of this report to do so. However, this report will include discussion of some empirical facts that illustrate the problems. The Little Theory of Planned Factor-time Supply (thereafter LTPFS) is designed to provide an alternative method of constructing the labor supply component of explanations of aggregate economic activity.

For every element of the class of distribution functions considered in this project, the conditional frequency function of L, given x, is defined as

(2.10) $f(L|x) = f(x, L)/f(x)$.

This conditional f.f. is the basis for construction of the LTPFS. It allows definition of the following conditional expectation (for the class of distributions considered here, the conditional expectation is always defined):

(2.11) $E(L|x) = \int_{-\infty}^{\infty} L\ f(L|x)dL$;

and, hence, of the conditional expectation

(2.12) $E(L|X)$.

The conditional expectation $E(L|X)$ may be thought of as the per capita planned supply of factor-time to the production and investment sector of the economy given X. Later in this report there will be specific functional forms assigned to (2.12). These will appear similar to functional forms of functions usually called "labor supply functions." Thus, conditional expectations like (2.12) are called "quasi-labor supply functions." It is here emphasized that these are not equivalent to the labor supply functions that appear in other studies. The prefix "quasi-" is meant to maintain an important distinction between the two types of functions.

An alternative way of expressing the labor supply function is to measure the amounts supplied in units of time, rather than in the value of product as in (2.12). This conversion is made by assigning

(2.13) $L = w \cdot T$,

where w is wage per unit of time measured as T. Then, an alternative quasi-labor supply function is

(2.14) $E(T|X) = (1/w)E(L|X)$

This alternative form is often used in classical discussions of both individual and per capita labor supply theories and empirical tests. It is also the

form usually used in presentations of theories explaining aggregate economic activity.

Specific Superpopulation Models

The first major step in applying the SPM approach to survey data is the proposal of specific functional forms for F(m, z, L, s) and for F(m_1, . . ., m_n, z, L, s). Naturally, the rights of freedom of assumption and discharge apply here. The economist is free to use whatever distribution functions meet the needs and goals of the research project. In deciding what functions to use, the economist may be guided by his knowledge of classical economic theory, as well as by empirical observations. These sources of knowledge are not limiting, however. Some of classical economic theory may not prove useful in applying the SPM approach.

There are two potential SP models that deserve brief mention here, not because they appear at present to be useful models, but because of their place in classical economic theory. The first model is the SPM that would result if the assumption of identical tastes and preferences required for perfect aggregation were met. The second is the model that would result if consumers exhibited maximum entropy, i. e., if each sample unit were equally likely to have any value in the domain of the variable measured. These two alternative models are at opposite ends of the scale measuring consumer-worker homogeneity. The model adopted for more complete study is presented in chapter III.

1. Uniformly Homogeneous Worker-Consumers

Uniform homogeneity of consumers was probably not intended as a permanent component of economic theory. It seems to have been used originally

as a means of overcoming certain obstacles in the way of testing hypotheses from classical consumer demand theory. That it has become a "stylized fact" seems to have occurred by accident. Nevertheless, it is worthwhile to ask what kind of two-step sampling process would apply if this assumption were accurate.

We temporarily introduce the assumption that the labor supply and consumption allocation decisions are separable. The consequence of this assumption for the SPM sampling process is that the conditional distribution of expenditures given total expenditure is a degenerate distribution in n-dimensional Euclidian space:

$$F(m_1, \ldots, m_n | m) = \begin{cases} 0, & (m_1, \ldots, m_n) < (A_1, \ldots, A_n) \text{ term-by-term} \\ 1, & (m_1, \ldots, m_n) \geq (A_1, \ldots, A_n) \text{ term-by-term.} \end{cases} \quad (2.15)$$

That similar analyses apply in many other cases is obvious. Examination of the homogeneity hypothesis in the framework of the SPM approach makes two points clear: (1) The SPM approach provides a method for testing the assumption and also shows that the assumption is rejected, and (2) the SPM approach does away with the necessity for using the homogeneity assumption, i.e., when using the SPM approach the obstacles that required the use of the assumption in other circumstances simply do not exist.

2. Maximum Entropy Superpopulations

Maximum entropy superpopulation models stand in opposition to models based on homogeneity of consumers. They also provide the simplest models in the SPM framework. Let $\mathcal{D}$ be the domain of $(m_1, \ldots, m_n, z, L, s)$. Then

the maximum entropy SPM assumes that each element of $\mathcal{D}$ is equally likely to appear as the measurement associated with a particular sample point. It also means, of course, that there is nothing to be gained by using any sampling design other than simple random sampling.

Characterization of the SP model associated with a maximum entropy superpopulation begins with the f.f.

$$(2.16)\qquad f(m_1, \ldots, m_n, z, L, s) = \begin{cases} K, & (m_1, \ldots, m_n, z, L, s) \varepsilon\, \mathcal{D} \\ 0 & \text{otherwise} \end{cases}$$

It is recognized that definition of $\mathcal{D}$ is a matter of choice, and that the choice of $\mathcal{D}$ determines the form of other f.f.'s that are derived as part of the SPM. These comments about $\mathcal{D}$ hold for other superpopulations as well. The domain used here is as follows:

$$\begin{aligned} \mathcal{D} = \{(m_1, \ldots, m_n, z, L, s):\ & m_i > 0,\ i = 1, \ldots, n; \\ (2.17)\qquad & m_1 + m_2 + \ldots, + m_n = m < z;\ 0 < y - (z + L + s); \\ & 0 < y < \infty;\ y \text{ is a parameter}\} \end{aligned}$$

This domain determines

$$(2.18)\qquad K = (n + 3)!/y^{n+3}.$$

The first step is to find the marginal joint f.f. of (m, z, L, s). It is

$$(2.19)\qquad h(m, z, L, s) = \begin{cases} Km^{n-1}/(n-1)! & 0 < m < z \\ & 0 < y - (z + L + s) \\ 0 & \text{otherwise} \end{cases}$$

The joint marginal f.f. of z, L, and s is:

$$(2.20)\qquad h_1(z, L, s) = (n + 3)!z^n/y^{n+3}n! \quad , \quad 0 < y-(z + L + s)$$

The conditional distribution of z, L, and s given $x = z + L + s$ is:

(2.21) $\quad g(z, L, s|x) = (n + 2)!z^n/n!x^{n+2}$, $\quad 0 < y-(z + L + s)$
$x = z + L + s.$

The conditional expectation of L given X is

(2.22) $\quad E(L|x) = x/(n + 3)$

(2.23) $\quad E(L|X) = (X - \lambda)/(n + 3).$

Finally, using (2.13)

(2.24) $\quad E(T|X) = (X - \lambda)/w(n + 3).$

Formula (2.24) is the quasi-labor supply function associated with the maximum entropy model.

The mean value of L is

(2.25) $\quad E(L) = y/n + 4$

and the d.f. of L is

(2.26) $\quad H_3(L) = 1 - (1 - L/y)^{n+3}$, $\; 0 < L < y$

This maximum entropy model constitutes the simplest hypothesis form of SPM used in this research. It contains only two parameters, λ and n. The parameters, λ, can be estimated in a variety of ways. One of the simplest methods of estimation is to estimate

(2.27) $\quad (\bar{T}_j/X_j) - [X_j/\bar{\omega}_j(n + 3)] = \lambda/\bar{W}_j\ (n + 3),\ j = 1, \ldots, k$

where the subscript j denotes the j^{th} strata of X for conditioning T, and $\bar{T}_j$ and $\bar{W}_j$ are the means of T and W for the j^{th} λ stratum.

The simplicity of this model would lead to selecting it as an <u>empirical hypothesis</u> independently of any assumptions about economic decision-making.

Rejection of this model as a suitable SPM would increase the burden and cost of further SPM research. Economies associated with this model justify testing it. However, it may be worthwhile to postulate decision criteria

and sampling conditions that could lead to such a model. Basmann (1980, sec. 3.1) gives one set of conditions sufficient to produce the maximum entropy model. Of course, there are others, but it does not seem necessary at this time to devise and discuss them.

Basic concepts of creating superpopulation models have been introduced, discussed and exemplified. Where the goal is to explain observed economic behavior as represented in survey samples, superpopulation techniques appear potentially powerful. The specific goal here is to suggest models and modeling techniques that can be used to study and explain consumption expenditures and supply of factor time. There are many potential superpopulation models. In the following chapters attention is focused on one that appears promising for further development.

III. BETA II SUPERPOPULATION MODELS FOR STUDYING LABOR SUPPLY BEHAVIOR

A. Introduction

In chapter II we introduced the SPM concept and examined several example SPMs and their implications for explaining consumer behavior. The objective of this section is presentation of the model selected for extensive study in this preliminary report. There are, of course, alternatives to the model presented here, but their development and exploitation must wait for increased time and research support. There is more than enough work on this one model within current time and resource constraints.

B. SPM Beta II

1. Population, Sample and Model

The finite population studied consists of the addresses appearing in the 1970 United States Census of Population. Sample data consist of observations on demographic and financial variables associated with consumer units (families or households) living at predetermined sample addresses drawn from the finite population. Details of sample design and procedures are found in chapter IV of this report. These sample data are referred to as the 1972-73 Consumer Expenditure Survey or CES.

From the extensive data contained in the sample, twenty-seven basic variables were selected to be studied and explained using the SPM approach. Complete specification of these variables is found in chapter IV and the documents cited therein. Here are the names and symbols associated with the variables:

Expenditure Categories: CES Interview
Sample, 1972-73

M_1 = Total Expenditures on Food

M_2 = Alcoholic Beverages

M_3 = Tobacco Products

M_4 = Shelter

M_5 = Fuel and Utilities

M_6 = Household Operations

M_7 = House Furnishings and Equipment

M_8 = Dry Cleaning and Laundry

M_9 = Clothing, Men, 16 Years and Over

M_{10} = Clothing, Boys, 2 through 15 Years

M_{11} = Clothing, Women, 16 Years and Over

M_{12} = Clothing, Girls, 2 through 15 Years

M_{13} = Clothing, Infants under 2

M_{14} = Clothing Materials, Clothing Repairs and Services

M_{15} = Transportation, Total (Excludes Recreational Vehicles, Vacation and Nonpleasure Trips)

M_{16} = Health Care

M_{17} = Personal Care

M_{18} = Recreation, Owned Vacation Home

M_{19} = Recreation, Vacation Trips

M_{20} = Recreation, Boats, Aircraft and Wheel Goods

M_{21} = Recreation, Other Recreation

M_{22} = Reading

M_{23} = Education

M_{24} = Miscellaneous Current Consumption Expenditures

$M = M_1 + M_2 + . . . + M_{24}$

$Z = M$ + Expenditures on Securities

Income Categories: CES Interview
Sample, 1972-73

L = Income from employment

S = Income from other sources (not including employment).

There are a variety of other observations on demographic and financial variables for consumer units in the CES. These twenty-seven variables have been selected for explanation in this early research effort. They are written in a single vector as $(M_1, M_2, . . ., M_{24}, Z, L, S)$. In this chapter a Superpopulation Model for these variables is proposed and explored.

2. Parameter and Variable Domains

Expenditures on commodities, $M_1, . . ., M_{24}$, and securities, $Z\text{-}M$, are of course non-negative. The same is true for income measures L and S. Consequently, the domain of $(M_1, . . ., M_{24}, Z, L, S)$ is

$$(3.1) \qquad \begin{aligned} M_i &\geqq 0 \qquad i = 1, . . ., 24 \\ Z\text{-}M &\geqq 0 \\ L &\geqq 0 \\ S &\geqq 0. \end{aligned}$$

These are the variables of the SPM.

The SPM chosen also contains parameters. The parameters and their domain are as follows:

$$(3.2) \qquad \begin{aligned} &a_i > 0 \qquad i = 1, . . ., 24 \\ &c_i > 0 \qquad i = 1, 2, 3 \\ &b - \sum_{i=1}^{24} a_i - (c_1 + c_2 + c_3) > 0 \end{aligned}$$

$$a = a_1 + a_2 + \ldots + a_{24}$$

$$c = c_1 + c_2 + c_3$$

$$K > 0$$

$$\lambda_i \gtreqless 0 \qquad i = 1, \ldots, 24$$

$$\lambda = \lambda_1 + \lambda_2 + \ldots + \lambda_{24}.$$

Call the parameter space Ω.

The parameters $\lambda_1, \lambda_2, \ldots, \lambda_{24}$ and λ are used for a translation of expenditure categories. Accordingly we define new variables as follows:

$$m_i = M_i - \lambda_i \qquad i = 1, \ldots, 24$$

(3.3) $$m = M - \lambda$$

$$z = Z - \lambda.$$

Taken together (3.1) and (3.2) constitute the applicable domain of the SPM developed here. Specification of this domain lays sufficient foundation for specification of the model.

3. SPM Beta II

The central focus of this report is use of a SPM to explain consumer and labor force behavior observed in the 1972-73 Consumer Expenditure Surveys. The model used to provide the explanation is called SPM Beta II. It consists of a multivariate distribution function with arguments $m_1, \ldots, m_{24}$, z, L, S and parameters $a_1, \ldots, a_{24}, c_1, c_2, c_3$, b and K. Rather than continually writing the somewhat complicated d.f. form of the model the frequency function (f.f.) of Beta II is used. The frequency function is as follows:

(3.4) $$f(m_1, \ldots, m_{24}, z, L, S) =$$

$$\frac{K^{b-a-c}\, m_1^{a_1-1}\, m_2^{a_2-1} \cdots m_{24}^{a_{24}-1}\, (z-m)^{c_1-1}\, L^{c_2-1}\, S^{c_3-1}}{B(a_1, \ldots, a_{24}, c_1, c_2, c_3, b-a-c)\, (K + z + L + S)^b}.$$

The domain for this function is given by (3.1)-(3.3). That domain is designated hereafter as D. The notation $f(\cdot) = f(m_1, \ldots, m_{24}, z, L, S)$ and $F(\cdot) = F(m_1, \ldots, m_{24}, z, L, S)$ will often be used to simplify and abbreviate the exposition.

The distribution function $F(\cdot)$ associated with the f.f. in (3.4) is called a Superpopulation Model for the twenty-seven variables selected from the Consumer Expenditure Survey. As a model of the population, it should accurately describe and explain the observed sample data. This particular model actually consists of a noncountable family of distribution functions each member of which is characterized by a definite collection of real numbers in place of the parameters in Ω.

A superpopulation model provides descriptions of patterns of economic behavior as well as methods for characterizing and studying the observed patterns. For example, the SPM can be used to calculate hypothetical proportions of population elements lying within specified subsets of the sample space. Various patterns of hypothetical proportions will be associated with alternative models, different populations and varying economic conditions. A superpopulation model completely and comprehensively characterizes the population. By so doing it provides a comprehensive explanatory model and specifies empirical testing procedures for examining the accuracy and applicability of the model. It goes well beyond the explanations provided by models that rely on only the first few population moments.

Included in the model are various submodels of subgroups of variables. Models for individual variables or subsets of variables are provided by marginal and conditional distribution functions derived from the main SPM.

Such models may allow for simpler and more elegant models where focus is on only a subset of population variables. In this present study of labor supply focus is consistently on a collection of four variables, m, z, L and S. Various submodels are employed that provide superpopulation models of these variables.

The marginal joint frequency function of m, z, L, and S is

$$(3.5) \qquad h(m, z, L, S) = \frac{K^{b-a-c}\, m^{a-1}\, (z-m)^{c_1-1}\, L^{c_2-1}\, S^{c_3-1}}{B(a, c_1, c_2, c_3, b-a-c)\, (K + z + L + S)^{b}} .$$

Examination of h(m, z, L, S) shows how the SPM characterizes the population. The population is characterized by the form of the frequency function, by the parameter values and by the parameter and variable domains. Summary measures, such as the Gini coefficient of concentration, may also be useful in characterizing the population.

Some of the parameters have direct economic interpretations; others have indirect interpretations. The parameter K has a direct economic interpretation. It is the minimum value of the sum Z + L + S. The sum Z + L + S is the total of economic variables to be allocated during the time period covered. Z is the sum of expenditures to be allocated, while L + S is the sum of disposable income to be allocated. Thus, the sum

$$(3.6) \qquad X = Z + L + S, \qquad x = X - \lambda$$

is the sum of expenditures and income to be allocated during a given time period. K is the minimum value of X in the population. It is a parameter to be estimated and has a direct economic interpretation. The composite variable X will come up for discussion and interpretation later in this chapter.

The parameters $a_1, \ldots, a_{24}$ and $a = a_1 + \ldots + a_{24}$ are associated with observed expenditures on commodity groups. An increase in a_i implies a relative increase of planned expenditures M_i. Likewise, an increase in $\underline{a}$ implies a relative shift toward increased expenditures on commodity bundles $M_1, \ldots, M_{24}$. Parameter c_1 is similarly associated with expenditures on securities as measured by Z-M.

Parameters c_2 and c_3 are associated with income variables L and S. An increase in c_2 implies a relative shift of allocation into L and an increase in c_3 implies a relative shift of allocation into S. Parameter $\underline{b}$ seems to be associated with the allocation process. As $\underline{b}$ increases, the distribution of Z + L + S seems to be spread more evenly. That is, it appears that increasing $\underline{b}$ increases apparent entropy in allocation of commodity expenditures and income.

What we wish to notice is that a specific collection of parameters $a_1, \ldots, a_n, c_1, c_2, c_3,$ b, and K identify a particular pattern of economic activity. They identify a particular structure as one of the collection of structures within the model Beta II. Thus, it can be seen that specifying a collection of parameters also identifies a pattern of economic activity. An extension of such pattern recognition is pattern prediction. One goal of this research was creation of a modeling system that would allow identification of consumer and worker responses to alternative economic constraints and conditions. This is partially fulfilled by creation of superpopulation models using distribution functions that characterize population behavior and response patterns directly through alternative parameter constellations. This is one of the major opportunities for further development afforded by the superpopulation modeling technique.

One definite population characteristic implied by the SPM Beta II is the joint determination of allocations of both income and expenditure variables. This model characterized a population where planned supply of factor time, planned other income, and planned expenditures are jointly selected and apparently influence one another. This property of Beta II shows up during examination of certain marginal density functions.

The joint marginal density function of labor income, L, and other income, S, is

$$(3.7) \qquad h(L, S) = \frac{K^{b-a-c} L^{c_2-1} S^{c_3-1}}{B(c_2, c_3, b-a-c)\ (K + L + S)^{b-a-c_1}} \quad \text{on } D$$

$$0 \qquad \text{elsewhere.}$$

Further, the marginal density function of labor income is

$$(3.8) \qquad h_L(L) = \frac{K^{b-a-c} L^{c_2-1}}{B(c_2, b-a-c)\ (K + L)^{b-a-c_1-c_3}} \quad \text{on } D$$

$$0 \qquad \text{elsewhere.}$$

Finally, the sum Y = L + S, income, has marginal density function

$$(3.9) \qquad h_Y(Y) = \frac{K^{b-a-c} Y^{c_2 + c_3 - 1}}{B(c_2 + c_3, b-a-c)\ (K + Y)^{b-a-c_1}} \quad \text{on } D$$

$$0 \qquad \text{elsewhere.}$$

Each of the density functions $h(L,S)$, $h_L(L)$ $h_Y(Y)$ is characterized by all of the parameters from the joint density function for SPM Beta II. The

density $h_Y(Y)$ is associated with the concepts from studies of income distributions. In particular, it has Gini coefficient equal to

$$(3.10) \qquad g_{Beta\ II} = \frac{2\ B(2(c_2 + c_3),\ 2(b-a-c)-1)}{(c_2 + c_3)\ [B(c_2 + c_3,\ b-a-c)]^2} \quad .$$

What is to be noted is this: the Gini coefficient and the income distribution are direct functions of the parameters characterizing consumption expenditures. Not only do the parameters for income variables, c_2 and c_3 affect the distribution of income, but so do the parameters for expenditures, $a_1, \ldots, a_n$, and the parameter for allocation, b.

Consumer unit populations are also characterized by their responses to economic conditions conditional upon some specific circumstances. Such responses are modeled by conditional distribution functions. These superpopulation models allow study of consumer and worker populations given specific values of some variables.

In this study the conditioning variable is x. While there are several variables that could be used as conditioning variables x is used here because of the focus on labor supply behavior. The marginal density function of x is

$$(3.11) \qquad h(x) = \begin{cases} \dfrac{K^{b-a-c}}{B(a + c,\ b-a-c)} \dfrac{x^{a+c-1}}{(K + x)^b} & K < x < \infty \\ 0 & \text{elsewhere .} \end{cases}$$

This function contains all of the parameters that characterize the population.

The marginal density of x is used to derive the pair of conditional densities studied. These are the conditional density of L given x and the

conditional density of $L + S = Y$ given x. The conditional density of L given x is

$$(3.12) \qquad g(L|x) = \begin{cases} \dfrac{L^{c_2-1}(x-L)^{a+c_1+c_3-1}}{B(a+c_1+c_3,\, c_2)\, x^{a+c-1}} & 0 < L < x < \infty \\ 0 & \text{elsewhere} \end{cases}$$

The conditional density of Y given x is

$$(3.13) \qquad g(Y|x) = \begin{cases} \dfrac{Y^{c_2+c_3-1}(x-Y)^{a+c_1-1}}{B(c_2+c_3,\, a+c_1)\, x^{a+c-1}} & 0 < Y < x < \infty \\ 0 & \text{elsewhere} \end{cases}$$

These densities model the distribution of labor income and total income for specific levels of the sum of income and expenditures to be allocated. For now they will be used primarily to characterize per capita behavior.

Per capita behavior is partially characterized by the conditional expectations

$$(3.14) \qquad E(L^k|x) = \frac{(c_2)_k}{(a+c)_k}\, x^k$$

$$(3.15) \qquad E(Y^k|x) = \frac{(c_2+c_3)_k}{(a+c)_k}\, x^k \quad .$$

The conditional expectation

$$(3.16) \qquad E(L|x) = \frac{c_2}{a+c}\, x$$

after translation becomes

$$(3.17) \qquad E(L|X) = \frac{c_2}{a+c}\,(X-\lambda).$$

If L is determined at a fixed hourly wage rate, so that

$$(3.18) \qquad L = w \cdot t_1$$

where w is the wage rate per hour and t_1 is the measure of hours worked, then

$$(3.19) \qquad E(t_1|X) = \frac{c_2}{a + c} (X-\lambda)/w.$$

The conditional expectation (3.19) is the per capita labor supply function in the Little Theory of Planned Factor-time Supply.

The labor supply function (3.19) has two important characteristics also shown by empirical per capita functions. It is downward sloping as a function of the wage rate and it is semi-parabolic in shape. Yates (1980) has shown that empirical per capita labor supply functions, using either time-series or cross-section data, have these properties. This contrasts sharply with the textbook treatments using upward sloping linear aggregate labor supply functions.

C. Summary

The model Beta II is presented as the initial model for study in the superpopulation modeling approach to explaining consumer and worker behavior observed in sample surveys. It is to be applied to and tested against the 1972-73 Consumer Expenditure Surveys. Estimation methods for Beta II are discussed in chapter IV and results are presented in chapter VI.

IV. ESTIMATION METHODS FOR SPM BETA II

A. Introduction

In this study it is important to have estimates both of parameters of the SPM and of the distribution function. Therefore, the method followed is that of first estimating the parameters and then using estimated parameter values to tabulate estimated distribution functions. This means that direct methods of estimating distribution and density functions bypassing estimates of parameters are not useful in the present context.

Creation of practicable and reliable estimation techniques for SPM Beta II is somewhat complicated by introduction of the population boundary values $\lambda_1, \ldots, \lambda_n$ and K. It is well known in univariate statistics that boundary value parameters make solution of likelihood or moment condition equations more complicated. Furthermore, the property of sufficiency is often lost when boundary parameters must be estimated; cf., Kendall and Stuart, Vol. II, chapters 18 and 23. In the present study these problems are mitigated to some extent by the very large sample sizes. Estimation techniques are adopted based on practical needs. The form of SPM Beta II models and others being studied shows the need for further development of statistical estimation and inference techniques in the presence of boundary value parameters.

In these early stages of research the boundary value parameters are estimated separately using order statistics. These estimates are then used in method of moment (MME) and maximum likelihood (MLE) formulas to obtain estimates of the other population parameters. Thus, the MME and MLE's are calculated conditionally upon the order statistic estimates of the boundary value parameters.

B. Estimating $\lambda_1, \ldots, \lambda_n$ and K

In economic parlance the parameters $\lambda_1, \ldots, \lambda_n$ are called minimum expenditures on commodity groups. K is accordingly called the minimum of the sum Z + L + S. These parameters are estimated by the corresponding minimum order statistics from the sample. For example, λ_1 is estimated by the minimum reported expenditures on food among all consumer units in the sample. Likewise for $\lambda_2, \ldots, \lambda_n$.

K is estimated as the minimum value of the sum Z + L + S across all consumer units in the sample. K could also be estimated as the sum of the sample minima for Z, L, and S. However, in this particular study it was decided to estimate K as the minimum of the sum. In later studies the sum of minima will also be used as an estimate of K. At the present time there do not appear to be any criteria for selecting one estimator of K over the other using either practical considerations or statistical properties.

Estimation of $\lambda_1, \ldots, \lambda_n$ and K lays foundation for estimation of the other parameters in Beta II, viz., $a_1, \ldots, a_n, c_1, c_2, c_3$, and b. The parameters $a_1, \ldots, a_n, c_1, c_2, c_3$ and b are estimated using either method of moments or maximum likelihood techniques conditional upon estimates of the boundary values. MME and MLE methods are applied given the previous estimates of $\lambda_1, \ldots, \lambda_n$ and K. Thus, their properties as conditional estimation techniques should be the same as those calculated using these techniques where there are no boundary value parameters.

C. Survey of Estimation Techniques

Estimates of $a_1, \ldots, a_n, c_1, c_2, c_3$, and b are obtained conditional upon estimates of $\lambda_1, \ldots, \lambda_n$ and K. That is, after estimation of

$\lambda_1, \ldots, \lambda_n$ and K the other parameters are estimated as if the boundary values are known with probability one. Discussion in the rest of this chapter rests on that procedure and is related to the conditional MME and MLE estimators.

There are a variety of parametric estimation techniques available to be used in estimating $a_1, \ldots, a_n$, c_1, c_2, c_3 and b. Two broad types of techniques considered for use are (a) maximum likelihood and (b) method of moments. In addition, these broad types are further subdivided according to whether they are applied to the joint distribution function or to marginal distribution functions for single variables. Those applied to the entire joint d.f. are called "system consistent" methods while those applied to marginal d.f.'s are called "single variable" methods.

The various conditional estimation techniques considered can be divided into four categories as shown in TABLE IV-1. The mnemonic designations in the four boxes in the table are names given to the four estimation procedures.

TABLE IV-1

	Single Variable	System Consistent
Maximum Likelihood	SVML	SCML
Method of Moments	SVMM	SCMM

The system consistent methods are so named because they provide consistest estimators for parameters in Beta II. Consistency of the maximum likelihood estimators follows since Beta II is a regular member of the multivariate exponential class. Method of moment estimator consistency follows since the

solution to the moment equations is unique, the property required for method of moment estimators to be consistent; cf. Rao (1965) pp. 287-288. Being consistent estimators is the main advantage of the system estimators over the single variable estimators.

The primary advantage of the single variable estimators is their simplicity for computations. Significantly smaller use of computing resources is made for single variable estimators. However, this advantage is largely eliminated by the rather simple system consistent method of moments (SCMM) estimators. The single variable methods do not appear to possess desirable statistical properties as compared with system consistent methods.

An alternative classification according to method of moments versus maximum likelihood methods suggests that method of moments estimators are preferred according to the simplicity criterion. The Beta II model contains multiple beta functions, $B(\alpha_1, \ldots, \alpha_k)$. Consequently, exact solutions to the likelihood equations are precluded and iterative approximation solutions must suffice. This suggests that exact solutions available using method of moments estimators are significantly simpler and less expensive to compute. These, briefly, are the reasons for choosing to focus exclusively on SCMM estimators in these early stages of the research.

D. System Consistent Method of Moments

The SCMM estimates employed are reasonably simple to calculate and provide parameter estimates that are consistent in the large sample sense. SCM Beta II as presently formulated has exactly one more parameter than it has variables; there is one parameter for each variable and one additional parameter associated with linear combinations of variables. This fact allows for

a direct method of calculating method of moment estimates. The equations to be solved for the SCMM estimates are as follows:

$$
(4.1)\qquad
\begin{aligned}
E(M_i) &= \bar{M}_i \qquad i = 1, \ldots, n \\
E(L) &= \bar{L} \\
E(Z) &= \bar{Z} \\
E(S) &= \bar{S} \\
|\Sigma| &= V_{M_1, \ldots, M_n, Z, L, S}
\end{aligned}
$$

Bar ($^{-}$) indicates sample mean value.

where $|\Sigma|$ is the generalized variance of SPM Beta II and $V_{M_1, \ldots, M_n, Z, L, S}$ is the sample generalized variance for the variables subscripted to V. Equations (4.1) are solved simultaneously to obtain estimates of the parameters $a_1, \ldots, a_n$, c_1, c_2, c_3 and b.

Solution to (4.1) will be unique if $|\Sigma|$ and $V_{M_1, \ldots, M_n, Z, L, S}$ are strictly positive. Where one or both of these values is not distinguishable from zero, the Beta II model is said to be based on a singular distribution. Existence of a unique solution to (4.1) implies that SCMM estimates are consistent. Of course the sample means are also unbiased for the corresponding population values. Other properties of the SCMM estimators, such as efficiency and sufficiency remain to be explored. It is recognized that SCML estimates would be sufficient due to the membership of Beta II in the multivariate regular exponential class. However, these sufficient statistics are functions of sums of logarithms of variables. There is apparently no evidence regarding sufficiency of SCMM available at this time.

E. Conclusions

The collection of parameters in the d.f. Beta II contains boundary value parameters ($\lambda_1, \ldots, \lambda_n$, K) along with population parameters

$(a_1, \ldots, a_n, c_1, c_2, c_3, b)$. Knowledge of estimation techniques and their properties for these parameters is minimal. During the early phases of this research the boundary value parameters are estimated by order statistics. The remaining parameters are then estimated by the method of moments given estimates of $(\lambda_1, \ldots, \lambda_n, K)$. These estimates of $(a_1, \ldots, a_n, \ldots, b)$ are at least conditionally consistent given the estimates of $(\lambda_1, \ldots, \lambda_n, K)$. It is conjectured (but not proved at this time) that the full set of estimates $(\lambda_1, \ldots, \lambda_n, K, a_1, \ldots, b)$ are also unconditionally consistent when calculated as described.

V. DATA SERIES

A. Introduction

Superpopulation models like Beta II are designed for use with survey samples. As SPMs are currently conceived and developed, sample sizes should be large enough to justify reliance on asymptotic properties of inference and estimation techniques. Previous discussions of estimators and their properties found in this report show the necessity of large samples for the multivariate models used in this study. Where sample sizes are sufficiently large, superpopulation models will allow detailed study of finite and conceptually infinite parent populations from which survey samples are drawn.

Four different data sets were considered for exploratory modeling using techniques proposed at the outset of this research. These were: (1) a set of economic data recorded as part of a series of token economy experiments; (2) a sample drawn from the Air Force Historical Personnel Data file and the Air Force Human Resources Laboratory, Brooks AFB, Texas; (3) 1972-73 Computer Expenditure Surveys; and (4) National Longitudinal Surveys, 1965-1976. Of these four potential data sources only (3) has proved to be amenable to analysis by the techniques reported herein. Data set (1) has sufficient detail on expenditures and incomes for consumer units. However, only six consumer units were involved in the experiment, so that there were too few observations to allow estimation of parameters and models. Data set (2) represents a finite population sufficiently large to host a large sample. But there is not enough information about or variation among consumer units to warrant the expense of drawing a sample from this population. Data set (4) consists of a sufficiently large finite sample. It was not used because,

like data set (2), it does not contain a sufficient amount of information regarding income sources and expenditures for individual consumer units. It may be useful in future research if study shows that decisions fixing supply of factor time, income by source, and expenditures can successfully be modeled separately from one another.

The remaining data set, (3) 1972-73 Consumer Expenditures Surveys, has sufficient large sample sizes and sufficiently detailed information to be modeled during this exploratory phase of research. It is discussed in some detail in the following sections.

B. Consumer Expenditure Survey Data

The data used to estimate the superpopulation model of labor supply and consumer expenditure called Beta II are the detailed interview data from the 1972-73 Consumer Expenditure Survey (CES). Various consumer expenditure surveys have been undertaken by the Federal government since 1888. A brief historical sketch of these surveys is found in Carlson's (1974) description of the 1972-73 CES. Carlson's description of the surveys provides an introduction to the survey data. There does not appear to be a single comprehensive source document and description of the CES surveys and data. In this section are presented the essential facts of the survey design.

The universe population consisted of the addresses contained in the 1970 Population Census. From this finite population were drawn two (apparently independent) samples of households, one for 1972 and one for 1973. Each household is designated as a "consumer unit." These samples were selected for visits by trained interviewers. The actual number of usable consumer responses obtained from the samples were 9,869 in 1972 and 10,106 in 1973.

Each consumer unit responding to the detailed interview supplied extensive demographic and financial information. Rights of privacy are preserved so that actual consumer units may not be identified. But consumer units still indicated family size, family composition and education, family expenditure patterns and family income by source. A more complete description of the survey data is found in Bulletin 1997 issued by the U. S. Department of Labor, Bureau of Labor Statistics.

In this analysis use is made of three basic types of information. Consumer unit expenditures on twenty-four basic commodity groups is modeled. Also, consumer unit income by source is modeled. Finally, consumer unit observations are subdivided in categories according to twelve occupation codes into which consumer units were grouped. Grouping was according to the occupation code for the head of the household. These three types of information are discussed in more detail in the following sections.

1. Expenditure Data

Expenditure categories used are defined according to those found in the documentation for the 1972-73 Consumer Expenditure Survey data tapes purchased from BLS. The relevant documentation is titled "1972-73 Consumer Expenditure Survey INTERVIEW SURVEY DETAILED PUBLIC USE TAPE," SECTION C (October 2, 1979) and SECTION D (October, 1977).

The twenty-four categories of expenditure used are drawn directly from the tape and are described in SECTION D. They are as follows:

Expenditure Categories: CES Interview Sample, 1972-73

M_1 = Total Expenditures on Food

M_2 = Alcoholic Beverages

M_3 = Tobacco Products

M_4 = Shelter

M_5 = Fuel and Utilities

M_6 = Household Operations

M_7 = House Furnishings and Equipment

M_8 = Dry Cleaning and Laundry

M_9 = Clothing, Men, 16 Years and Over

M_{10} = Clothing, Boys, 2 through 15 Years

M_{11} = Clothing, Women 16 Years and Over

M_{12} = Clothing, Girls, 2 through 15 Years

M_{13} = Clothing, Infants under 2

M_{14} = Clothing Materials, Clothing Repairs and Services

M_{15} = Transportation, Total (Excludes Recreational Vehicles, Vacation and Nonpleasure Trips)

M_{16} = Health Care

M_{17} = Personal Care

M_{18} = Recreation, Owned Vacation Home

M_{19} = Recreation, Vacation Trips

M_{20} = Recreation, Boats, Aircraft and Wheel Goods

M_{21} = Recreation, Other Recreation

M_{22} = Reading

M_{23} = Education

M_{24} = Miscellaneous Current Consumption Expenditures

An additional expenditure variable was created and used. It is called "expenditure on securities" and is specified by the formula

$Z = M_i + . . . + M_{24}$ + [Purchase price of any stocks, mutual funds or bonds (30196) + Reduction of Mortgage Principal (present home purchased before survey year) (31001) + Reduction of Mortgage Principle (previous home owned in survey year) (31002) + Reduction of principal (owned vacation home) (negative) (31005) + Reduction of principal (unimproved land) (negative) (31007) + Reduction of principal (other properties) (31009).]

The variable Z-M in square brackets is expenditure on securities.

In addition to these twenty-five expenditure variables we treat two sources of income as discussed in the next section.

2. Income Data

Two income variables were created from basic survey data for use in estimating Beta II. They are designed to reflect income from the sale of factor time, or labor income, and income derived from other sources, including asset income, transitory income, and transfer payments. The two income variables are designed as follows:

L = Civilian wages and salaries (27001) + Value of rent received as civilian pay (27004) + Value of food received as civilian pay (27005) + Armed forces pay on active or reserve duty (27006) + Armed forces quarters and subsistence allowance (27007).

S = Interest from bonds, saving accounts, loans and mortgages (27012) + Dividends from stocks (27013) + Dividends from mutual funds

(27014) + Other dividends (27015) + Income from social security and railroad retirement (27016) + Federal civil service retirement income (27017) + State and local government retirement income (27018) + Income from private pensions, annuities and retirement (27019) + Income from veterans compensation and benefits (27020) + Income from government unemployment insurance (27021) + Income from private unemployment insurance (27022) + Public assistance or welfare (27023) + Regular contributions for support (27024) + Other money income including workmen's compensation (27025) + Income from subleasing (27026) + Insurance refunds (27029) + Property tax refunds (27032) + Refund for overpayment payment of social security taxes (27033) + Federal tax refunds (27034) + State and local tax refunds (27035) + Other tax refunds (27036) + Inheritances or bequests received (29001) + Cash gifts or lump sum settlements received (29002).

Current indications are that the modeling is improved by separating components of S into subcategories such as asset income, transfer payments, and income from enterprises. This will be done in future studies.

As part of this research, alternative models are estimated for different occupational groupings. The occupation groupings assigned in the 1972-73 CES are discussed in the next section.

3. Occupational Grouping Codes

Consumer units surveyed were each assigned an occupational code based on the occupation of the household head. Occupation codes are also assigned to other employed members of consumer units and recorded in the survey data.

Here consumer units are sorted by occupation category for the consumer unit head and Beta II models estimated for each occupation code and each year.

Occupation codes and corresponding descriptions as applied in the CES are as follows:

Occupation Code	Description
01	self employed, including farm operators
02	salaried professional, technical and kindred workers
03	salaried managers and administrators, and kindred workers
	wage and other salaried:
04	clerical workers
05	sales workers
06	craftsmen
07	operatives
08	unskilled laborers, including household workers
09	service workers
10	not working, not retired
11	retired
12	other (in armed forces living off post, working without pay, N.R.)

Note that occupation code 12 represents a composite category. An extra sort was made for consumer unit heads with occupation codes equal to 12. For every occupation code 12 consumer unit a check was made to see whether that consumer unit had income from armed forces pay. Any consumer unit in occupation code without income from armed forces pay was eliminated from the

sample. This is to eliminate all but armed forces personnel living off post from occupation code 12. Thus, occupation code 12 represents, as far as possible, an occupation code for armed forces personnel.

C. Selection of Sample Points for Fitting Beta II

Interviews for the detailed interview tapes were conducted quarterly during 1972 and 1973. Survey forms and interview procedures were designed to collect annualized income and expenditure data for consumer units. Two problems were encountered in meeting this objective. First, some consumer units refused to report, or for some reason did not report, annual income. Second, some consumer units were not included in the survey for the entire year. For each consumer unit there is indication on the survey form (and on the data tapes) whether the reported income is full annual income and whether the consumer unit was included in the survey for a full year.

The decision was made to use only observations where the consumer unit reported full income and was also included in the entire year of the survey. All consumer units that did not show both full income reporting and a full year in the survey were eliminated from the sample used in estimating Beta II. In addition, for consumer units with occupation code 12 the restrictions discussed previously were imposed after the checks for full income and full year reporting. Model estimates reported in chapter VI are numbers of acceptable consumer units upon which estimates are based.

VI. ESTIMATION RESULTS FOR BETA II

A. Introduction

This chapter contains estimates of the parameters in Beta II. Parameter estimates are presented for most of the twelve occupation codes for both 1972 and 1973. In addition, parameter estimates for both survey years are calculated for the entire collection of acceptable observations and occupation codes. These parameter estimates are obtained at reasonable cost and show the feasibility of estimating SPM Beta II.

B. Estimates by Occupation Code

CES data were used to calculate SCMM estimates of all parameters in λ. Estimates were calculated for each occupation code for both 1972 and 1973 except where the sample covariance matrix was singular. This occurred in three cases: (1) occupation code 10 for 1972, (2) occupation code 10 for 1973, and (3) occupation code 12 for 1973. The singularity of the sample covariance matrix in these cases apparently occurs because recorded expenditures on certain commodity groups are zero. Estimates were calculated for the other twenty-one combinations of twelve occupation codes and two years.

Alternative estimation methods have been developed for dealing with the instances where the sample covariance matrix is singular. However, full development of these techniques and computer programming to implement them is beyond the scope and budget of this present project.

Estimates of λ_i were calculated first and used in the remaining calculations as described in chapter V.

Most estimates of λ_i were zero. Those that were not zero are presented in TABLE VI-1. All other values were estimated as zero using the minimum order statistics.

TABLE IV-1

Estimates of λ_i by Occupation Code
(Only Nonzero Estimates are Presented)

Occupation Code	1972 $\hat{\lambda}_1$	1973 $\hat{\lambda}_1$
5		2.80
7		1.86
8		2.20
3	13.70	
5	8.14	
6	2.70	
8	2.09	

Estimates of $\hat{K}$, $\hat{a}_1$, ..., $\hat{a}_{24}$, $\hat{c}_1$, $\hat{c}_2$, $\hat{c}_3$ and b are presented in TABLES VI-1.1972.1 through VI-1.1973.11. These estimates are calculated using SCMM.

TABLE VI-1.1972.1

Estimates of Parameters for Beta II

Occupation Code: 1

Number of Observations: 452

$\hat{K} = 947.24$	$\hat{c}_1 = 3.3941$
$\hat{a} = 26.7496$	$\hat{c}_2 = 6.6233$
$\hat{b} = 45.5616$	$\hat{c}_3 = 5.0881$
$\hat{a}_1 = 5.3943$	$\hat{a}_{13} = 0.0331$
$\hat{a}_2 = 0.2242$	$\hat{a}_{14} = 0.0999$
$\hat{a}_3 = 0.3885$	$\hat{a}_{15} = 5.1037$
$\hat{a}_4 = 4.0697$	$\hat{a}_{16} = 1.7754$
$\hat{a}_5 = 1.4461$	$\hat{a}_{17} = 0.3724$
$\hat{a}_6 = 1.0380$	$\hat{a}_{18} = 0.0689$
$\hat{a}_7 = 1.2863$	$\hat{a}_{19} = 0.8785$
$\hat{a}_8 = 0.2523$	$\hat{a}_{20} = 0.3378$
$\hat{a}_9 = 0.6179$	$\hat{a}_{21} = 1.0051$
$\hat{a}_{10} = 0.1232$	$\hat{a}_{22} = 0.1700$
$\hat{a}_{11} = 0.9652$	$\hat{a}_{23} = 0.5298$
$\hat{a}_{12} = 0.1879$	$\hat{a}_{24} = 0.3815$

Labor Supply Function (3.17): $E(L|X) = .1532X$

TABLE VI-1.1972.2

Estimates of Parameters for Beta II

Occupation Code: 2

Number of Observations: 1035

$\hat{K} = 2496.61$	$\hat{c}_1 = 1.9655$
$\hat{a} = 26.2055$	$\hat{c}_2 = 38.9096$
$\hat{b} = 78.0043$	$\hat{c}_3 = 3.4697$
$\hat{a}_1 = 4.7221$	$\hat{a}_{13} = 0.0358$
$\hat{a}_2 = 0.2724$	$\hat{a}_{14} = 0.1012$
$\hat{a}_3 = 0.2752$	$\hat{a}_{15} = 5.0573$
$\hat{a}_4 = 4.8447$	$\hat{a}_{16} = 1.3660$
$\hat{a}_5 = 1.1423$	$\hat{a}_{17} = 0.2894$
$\hat{a}_6 = 1.0687$	$\hat{a}_{18} = 0.0359$
$\hat{a}_7 = 1.3497$	$\hat{a}_{19} = 1.1377$
$\hat{a}_8 = 0.2483$	$\hat{a}_{20} = 0.1979$
$\hat{a}_9 = 0.6165$	$\hat{a}_{21} = 1.1413$
$\hat{a}_{10} = 0.1391$	$\hat{a}_{22} = 0.2184$
$\hat{a}_{11} = 0.8712$	$\hat{a}_{23} = 0.6365$
$\hat{a}_{12} = 0.1720$	$\hat{a}_{24} = 0.2653$

Labor Supply Function (3.17): $E(L|X) = .5515X$

TABLE VI-1.1972.3

Estimates of Parameters for Beta II

Occupation Code: 3

Number of Observations: 856

$\hat{K} = 3320.3$	$\hat{c}_1 = 2.7636$
$\hat{a} = 22.1891$	$\hat{c}_2 = 34.5633$
$\hat{b} = 69.9946$	$\hat{c}_3 = 3.1889$
$\hat{a}_1 = 4.3335$	$\hat{a}_{13} = 0.0328$
$\hat{a}_2 = 0.2232$	$\hat{a}_{14} = 0.0881$
$\hat{a}_3 = 0.3016$	$\hat{a}_{15} = 3.9899$
$\hat{a}_4 = 3.5612$	$\hat{a}_{16} = 1.2771$
$\hat{a}_5 = 1.0063$	$\hat{a}_{17} = 0.2997$
$\hat{a}_6 = 0.8387$	$\hat{a}_{18} = 0.0561$
$\hat{a}_7 = 1.1441$	$\hat{a}_{19} = 0.9572$
$\hat{a}_8 = 0.2293$	$\hat{a}_{20} = 0.3674$
$\hat{a}_9 = 0.6519$	$\hat{a}_{21} = 0.8975$
$\hat{a}_{10} = 0.1407$	$\hat{a}_{22} = 0.1457$
$\hat{a}_{11} = 0.8266$	$\hat{a}_{23} = 0.4459$
$\hat{a}_{12} = 0.1589$	$\hat{a}_{24} = 0.2156$

Labor Supply Function (3.17): $E(L|X) = .5512\ (X-13.7)$

TABLE VI-1.1972.4

Estimates of Parameters for Beta II

Occupation Code: 4

Number of Observations: 645

$\hat{K} = 2319.40$	$\hat{c}_1 = 0.8576$
$\hat{a} = 21.3690$	$\hat{c}_2 = 28.1120$
$\hat{b} = 62.5055$	$\hat{c}_3 = 4.3253$
$\hat{a}_1 = 4.2904$	$\hat{a}_{13} = 0.0270$
$\hat{a}_2 = 0.1681$	$\hat{a}_{14} = 0.0910$
$\hat{a}_3 = 0.3236$	$\hat{a}_{15} = 4.1127$
$\hat{a}_4 = 3.8523$	$\hat{a}_{16} = 1.3202$
$\hat{a}_5 = 0.9764$	$\hat{a}_{17} = 0.3143$
$\hat{a}_6 = 0.8425$	$\hat{a}_{18} = 0.0094$
$\hat{a}_7 = 1.0027$	$\hat{a}_{19} = 0.6853$
$\hat{a}_8 = 0.2470$	$\hat{a}_{20} = 0.1046$
$\hat{a}_9 = 0.3997$	$\hat{a}_{21} = 0.7808$
$\hat{a}_{10} = 0.1155$	$\hat{a}_{22} = 0.1509$
$\hat{a}_{11} = 0.9436$	$\hat{a}_{23} = 0.2423$
$\hat{a}_{12} = 0.1373$	$\hat{a}_{24} = 0.2314$

Labor Supply Function (3.17): $E(L|X) = .5143X$

TABLE VI-1.1972.5

Estimates of Parameters for Beta II

Occupation Code: 5

Number of Observations: 282

$\hat{K} = 2215.44$	$\hat{c}_1 = 2.2136$
$\hat{a} = 23.8787$	$\hat{c}_2 = 31.5522$
$\hat{b} = 69.4063$	$\hat{c}_3 = 4.9578$
$\hat{a}_1 = 4.9814$	$\hat{a}_{13} = 0.0323$
$\hat{a}_2 = 0.2459$	$\hat{a}_{14} = 0.0901$
$\hat{a}_3 = 0.3802$	$\hat{a}_{15} = 4.2235$
$\hat{a}_4 = 3.9808$	$\hat{a}_{16} = 1.4722$
$\hat{a}_5 = 1.1555$	$\hat{a}_{17} = 0.3212$
$\hat{a}_6 = 0.9802$	$\hat{a}_{18} = 0.0186$
$\hat{a}_7 = 1.1445$	$\hat{a}_{19} = 0.8155$
$\hat{a}_8 = 0.3078$	$\hat{a}_{20} = 0.1664$
$\hat{a}_9 = 0.6985$	$\hat{a}_{21} = 0.9571$
$\hat{a}_{10} = 0.1308$	$\hat{a}_{22} = 0.1626$
$\hat{a}_{11} = 0.8286$	$\hat{a}_{23} = 0.3930$
$\hat{a}_{12} = 0.1693$	$\hat{a}_{24} = 0.2228$

Labor Supply Function (3.17): $E(L|X) = .5040\ (X-8.14)$

TABLE VI-1.1972.6

Estimates of Parameters for Beta II

Occupation Code: 6

Number of Observations: 1268

$\hat{K} = 2770.15$	$\hat{c}_1 = 0.8941$
$\hat{a} = 25.1534$	$\hat{c}_2 = 35.6778$
$\hat{b} = 73.6092$	$\hat{c}_3 = 3.0295$
$\hat{a}_1 = 5.5466$	$\hat{a}_{13} = 0.0356$
$\hat{a}_2 = 0.2597$	$\hat{a}_{14} = 0.0985$
$\hat{a}_3 = 0.5051$	$\hat{a}_{15} = 5.8479$
$\hat{a}_4 = 3.2918$	$\hat{a}_{16} = 1.5165$
$\hat{a}_5 = 1.2802$	$\hat{a}_{17} = 0.3069$
$\hat{a}_6 = 0.7288$	$\hat{a}_{18} = 0.0260$
$\hat{a}_7 = 1.2154$	$\hat{a}_{19} = 0.7407$
$\hat{a}_8 = 0.2083$	$\hat{a}_{20} = 0.3658$
$\hat{a}_9 = 0.5453$	$\hat{a}_{21} = 0.8946$
$\hat{a}_{10} = 0.1747$	$\hat{a}_{22} = 0.1364$
$\hat{a}_{11} = 0.6735$	$\hat{a}_{23} = 0.2786$
$\hat{a}_{12} = 0.1857$	$\hat{a}_{24} = 0.2908$

Labor Supply Function (3.17): $E(L|X) = .5510\ (X-2.7)$

TABLE VI-1.1972.7

Estimates of Parameters for Beta II

Occupation Code: 7

Number of Observations: 1081

$\hat{K} = 1817.76$	$\hat{c}_1 = 0.7237$
$\hat{a} = 23.0573$	$\hat{c}_2 = 31.3111$
$\hat{b} = 64.5681$	$\hat{c}_3 = 2.9016$
$\hat{a}_1 = 5.2366$	$\hat{a}_{13} = 0.0470$
$\hat{a}_2 = 0.2322$	$\hat{a}_{14} = 0.0730$
$\hat{a}_3 = 0.5008$	$\hat{a}_{15} = 5.4713$
$\hat{a}_4 = 3.3460$	$\hat{a}_{16} = 1.1800$
$\hat{a}_5 = 1.2190$	$\hat{a}_{17} = 0.2910$
$\hat{a}_6 = 0.7206$	$\hat{a}_{18} = 0.0211$
$\hat{a}_7 = 1.0026$	$\hat{a}_{19} = 0.4825$
$\hat{a}_8 = 0.2330$	$\hat{a}_{20} = 0.2860$
$\hat{a}_9 = 0.4434$	$\hat{a}_{21} = 0.7881$
$\hat{a}_{10} = 0.1716$	$\hat{a}_{22} = 0.1128$
$\hat{a}_{11} = 0.6225$	$\hat{a}_{23} = 0.1971$
$\hat{a}_{12} = 0.1752$	$\hat{a}_{24} = 0.2040$

Labor Supply Function (3.17): $E(L|X) = .5399X$

TABLE VI-1.1972.8

Estimates of Parameters for Beta II

Occupation Code: 8

Number of Observations: 488

$\hat{K}$ = 589.96	$\hat{c}_1$ = 0.9578
$\hat{a}$ = 26.1255	$\hat{c}_2$ = 29.1333
$\hat{b}$ = 65.3491	$\hat{c}_3$ = 5.5993
$\hat{a}_1$ = 6.2529	$\hat{a}_{13}$ = 0.0597
$\hat{a}_2$ = 0.2797	$\hat{a}_{14}$ = 0.0849
$\hat{a}_3$ = 0.5800	$\hat{a}_{15}$ = 5.6173
$\hat{a}_4$ = 3.8742	$\hat{a}_{16}$ = 1.5116
$\hat{a}_5$ = 1.4004	$\hat{a}_{17}$ = 0.3234
$\hat{a}_6$ = 0.8310	$\hat{a}_{18}$ = 0.0151
$\hat{a}_7$ = 1.0831	$\hat{a}_{19}$ = 0.5329
$\hat{a}_8$ = 0.3291	$\hat{a}_{20}$ = 0.2760
$\hat{a}_9$ = 0.5680	$\hat{a}_{21}$ = 0.8659
$\hat{a}_{10}$ = 0.1936	$\hat{a}_{22}$ = 0.1167
$\hat{a}_{11}$ = 0.7064	$\hat{a}_{23}$ = 0.2005
$\hat{a}_{12}$ = 0.1996	$\hat{a}_{24}$ = 0.2234

Labor Supply Function (3.17): $E(L|X) = .4713\ (X-2.09)$

TABLE VI-1.1972.9

Estimates of Parameters for Beta II

Occupation Code: 9

Number of Observations: 613

$\hat{K} = 1121.4$	$\hat{c}_1 = 0.9837$
$\hat{a} = 24.6922$	$\hat{c}_2 = 27.8934$
$\hat{b} = 63.4861$	$\hat{c}_3 = 4.8609$
$\hat{a}_1 = 5.6098$	$\hat{a}_{13} = 0.0261$
$\hat{a}_2 = 0.2107$	$\hat{a}_{14} = 0.0720$
$\hat{a}_3 = 0.5121$	$\hat{a}_{15} = 5.1933$
$\hat{a}_4 = 4.0583$	$\hat{a}_{16} = 1.3939$
$\hat{a}_5 = 1.2949$	$\hat{a}_{17} = 0.3245$
$\hat{a}_6 = 0.8744$	$\hat{a}_{18} = 0.0204$
$\hat{a}_7 = 1.0200$	$\hat{a}_{19} = 0.6411$
$\hat{a}_8 = 0.3081$	$\hat{a}_{20} = 0.1626$
$\hat{a}_9 = 0.4925$	$\hat{a}_{21} = 0.8072$
$\hat{a}_{10} = 0.1489$	$\hat{a}_{22} = 0.1136$
$\hat{a}_{11} = 0.7820$	$\hat{a}_{23} = 0.2788$
$\hat{a}_{12} = 0.1486$	$\hat{a}_{24} = 0.1983$

Labor Supply Function (3.17): $E(L|X) = .4774X$

TABLE VI-1.1972.11

Estimates of Parameters for Beta II

Occupation Code: 11

Number of Observations: 1367

$\hat{K} = 785.19$	$\hat{c}_1 = 0.4832$
$\hat{a} = 14.6132$	$\hat{c}_2 = 2.4564$
$\hat{b} = 35.9648$	$\hat{c}_3 = 14.7612$
$\hat{a}_1 = 3.6401$	$\hat{a}_{13} = 0.0103$
$\hat{a}_2 = 0.1040$	$\hat{a}_{14} = 0.0391$
$\hat{a}_3 = 0.2032$	$\hat{a}_{15} = 2.1773$
$\hat{a}_4 = 2.4547$	$\hat{a}_{16} = 1.4391$
$\hat{a}_5 = 1.0714$	$\hat{a}_{17} = 0.2535$
$\hat{a}_6 = 0.7242$	$\hat{a}_{18} = 0.0195$
$\hat{a}_7 = 0.4807$	$\hat{a}_{19} = 0.5518$
$\hat{a}_8 = 0.1568$	$\hat{a}_{20} = 0.0483$
$\hat{a}_9 = 0.1909$	$\hat{a}_{21} = 0.3737$
$\hat{a}_{10} = 0.0062$	$\hat{a}_{22} = 0.0979$
$\hat{a}_{11} = 0.4193$	$\hat{a}_{23} = 0.0312$
$\hat{a}_{12} = 0.0101$	$\hat{a}_{24} = 0.1098$

Labor Supply Function (3.17): $E(L|X) = .0760X$

TABLE VI-1.1972.12

Estimates of Parameters for Beta II

Occupation Code: 12

Number of Observations: 115

$\hat{K} = 4068.73$	$\hat{c}_1 = 1.6607$
$\hat{a} = 30.8120$	$\hat{c}_2 = 36.2684$
$\hat{b} = 85.3036$	$\hat{c}_3 = 2.8882$
$\hat{a}_1 = 4.1492$	$\hat{a}_{13} = 0.0959$
$\hat{a}_2 = 0.3706$	$\hat{a}_{14} = 0.1120$
$\hat{a}_3 = 0.3039$	$\hat{a}_{15} = 7.6748$
$\hat{a}_4 = 6.5773$	$\hat{a}_{16} = 0.4070$
$\hat{a}_5 = 1.0309$	$\hat{a}_{17} = 0.3362$
$\hat{a}_6 = 1.2713$	$\hat{a}_{18} = 0.0103$
$\hat{a}_7 = 1.7417$	$\hat{a}_{19} = 0.9633$
$\hat{a}_8 = 0.4137$	$\hat{a}_{20} = 1.0101$
$\hat{a}_9 = 0.6918$	$\hat{a}_{21} = 1.5203$
$\hat{a}_{10} = 0.1849$	$\hat{a}_{22} = 0.2640$
$\hat{a}_{11} = 0.9809$	$\hat{a}_{23} = 0.1179$
$\hat{a}_{12} = 0.2256$	$\hat{a}_{24} = 0.3577$

Labor Supply Function (3.17): $E(L|X) = .5063\ (X-2.08)$

TABLE VI-1.1973.1

Estimates of Parameters for Beta II

Occupation Code: 1

Number of Observations: 380

$\hat{K}$ = 867	c_1 = 4.8117
$\hat{a}$ = 31.2490	c_2 = 7.4224
$\hat{b}$ = 51.9175	c_3 = 5.0583
$\hat{a}_1$ = 6.4815	$\hat{a}_{13}$ = 0.0690
$\hat{a}_2$ = 0.3471	$\hat{a}_{14}$ = 0.1118
$\hat{a}_3$ = 0.4075	$\hat{a}_{15}$ = 6.0004
$\hat{a}_4$ = 4.4110	$\hat{a}_{16}$ = 2.0773
$\hat{a}_5$ = 1.5824	$\hat{a}_{17}$ = 0.4007
$\hat{a}_6$ = 1.1999	$\hat{a}_{18}$ = 0.0751
$\hat{a}_7$ = 1.6206	$\hat{a}_{19}$ = 1.1075
$\hat{a}_8$ = 0.2481	$\hat{a}_{20}$ = 0.3270
$\hat{a}_9$ = 0.7236	$\hat{a}_{21}$ = 1.3519
$\hat{a}_{10}$ = 0.2035	$\hat{a}_{22}$ = 0.2015
$\hat{a}_{11}$ = 1.0167	$\hat{a}_{23}$ = 0.6483
$\hat{a}_{12}$ = 0.3150	$\hat{a}_{24}$ = 0.3216

Labor Supply Function (3.17): $E(L|X) = .1529X$

TABLE VI-1.1973.2

Estimates of Parameters for Beta II

Occupation Code: 2

Number of Observations: 1089

$\hat{K} = 1275.00$	$\hat{c}_1 = 2.7044$
$\hat{a} = 30.4646$	$\hat{c}_2 = 44.5259$
$\hat{b} = 88.2934$	$\hat{c}_3 = 6.0189$
$\hat{a}_1 = 5.6954$	$\hat{a}_{13} = 0.0646$
$\hat{a}_2 = 0.2922$	$\hat{a}_{14} = 0.1191$
$\hat{a}_3 = 0.2880$	$\hat{a}_{15} = 5.7694$
$\hat{a}_4 = 5.5672$	$\hat{a}_{16} = 1.5707$
$\hat{a}_5 = 1.3746$	$\hat{a}_{17} = 0.3187$
$\hat{a}_6 = 1.1961$	$\hat{a}_{18} = 0.0407$
$\hat{a}_7 = 1.6830$	$\hat{a}_{19} = 1.2318$
$\hat{a}_8 = 0.2516$	$\hat{a}_{20} = 0.3792$
$\hat{a}_9 = 0.6798$	$\hat{a}_{21} = 1.3592$
$\hat{a}_{10} = 0.1575$	$\hat{a}_{22} = 0.2489$
$\hat{a}_{11} = 0.9795$	$\hat{a}_{23} = 0.7129$
$\hat{a}_{12} = 0.1961$	$\hat{a}_{24} = 0.2883$

Labor Supply Function (3.17): $E(L|X) = .5319X$

TABLE VI-1.1973.3

Estimates of Parameters for Beta II

Occupation Code: 3

Number of Observations: 1030

$\hat{K} = 841.16$	$\hat{c}_1 = 3.5601$
$\hat{a} = 34.6123$	$\hat{c}_2 = 49.0040$
$\hat{b} = 96.5023$	$\hat{c}_3 = 5.9440$
$\hat{a}_1 = 6.9132$	$\hat{a}_{13} = 0.0646$
$\hat{a}_2 = 0.3795$	$\hat{a}_{14} = 0.1253$
$\hat{a}_3 = 0.4436$	$\hat{a}_{15} = 6.6138$
$\hat{a}_4 = 5.4499$	$\hat{a}_{16} = 1.9859$
$\hat{a}_5 = 1.5890$	$\hat{a}_{17} = 0.4402$
$\hat{a}_6 = 1.3456$	$\hat{a}_{18} = 0.0601$
$\hat{a}_7 = 1.8025$	$\hat{a}_{19} = 1.3065$
$\hat{a}_8 = 0.3162$	$\hat{a}_{20} = 0.5768$
$\hat{a}_9 = 0.9228$	$\hat{a}_{21} = 1.3719$
$\hat{a}_{10} = 0.2104$	$\hat{a}_{22} = 0.2385$
$\hat{a}_{11} = 1.2258$	$\hat{a}_{23} = 0.6087$
$\hat{a}_{12} = 0.2416$	$\hat{a}_{24} = 0.3797$

Labor Supply Function (3.17): $E(L|X) = .5262X$

TABLE VI-1.1973.4

Estimates of Parameters for Beta II

Occupation Code: 4

Number of Observations: 667

$\hat{K} = 1566.51$	$\hat{c}_1 = 1.0804$
$\hat{a} = 26.5711$	$\hat{c}_2 = 33.8376$
$\hat{b} = 73.4662$	$\hat{c}_3 = 5.6632$
$\hat{a}_1 = 5.6051$	$\hat{a}_{13} = 0.0427$
$\hat{a}_2 = 0.2189$	$\hat{a}_{14} = 0.0944$
$\hat{a}_3 = 0.4022$	$\hat{a}_{15} = 5.5308$
$\hat{a}_4 = 4.8771$	$\hat{a}_{16} = 1.5101$
$\hat{a}_5 = 1.2083$	$\hat{a}_{17} = 0.3576$
$\hat{a}_6 = 1.0418$	$\hat{a}_{18} = 0.0042$
$\hat{a}_7 = 1.1117$	$\hat{a}_{19} = 0.7493$
$\hat{a}_8 = 0.3230$	$\hat{a}_{20} = 0.1172$
$\hat{a}_9 = 0.4269$	$\hat{a}_{21} = 0.9223$
$\hat{a}_{10} = 0.1634$	$\hat{a}_{22} = 0.1649$
$\hat{a}_{11} = 1.0703$	$\hat{a}_{23} = 0.2567$
$\hat{a}_{12} = 0.1628$	$\hat{a}_{24} = 0.2093$

Labor Supply Function (3.17): $E(L|X) = .5039X$

TABLE VI-1.1973.5

Estimates of Parameters for Beta II

Occupation Code: 5

Number of Observations: 356

$\hat{K} = 2280.18$	$\hat{c}_1 = 1.9986$
$\hat{a} = 24.8099$	$\hat{c}_2 = 31.6361$
$\hat{b} = 69.9799$	$\hat{c}_3 = 5.1050$
$\hat{a}_1 = 4.9781$	$\hat{a}_{13} = 0.0461$
$\hat{a}_2 = 0.2865$	$\hat{a}_{14} = 0.0920$
$\hat{a}_3 = 0.3537$	$\hat{a}_{15} = 4.4541$
$\hat{a}_4 = 4.2850$	$\hat{a}_{16} = 1.4868$
$\hat{a}_5 = 1.1143$	$\hat{a}_{17} = 0.3177$
$\hat{a}_6 = 0.8957$	$\hat{a}_{18} = 0.0258$
$\hat{a}_7 = 1.4728$	$\hat{a}_{19} = 0.7544$
$\hat{a}_8 = 0.2473$	$\hat{a}_{20} = 0.3970$
$\hat{a}_9 = 0.7260$	$\hat{a}_{21} = 1.0137$
$\hat{a}_{10} = 0.1393$	$\hat{a}_{22} = 0.1471$
$\hat{a}_{11} = 0.8121$	$\hat{a}_{23} = 0.3437$
$\hat{a}_{12} = 0.1765$	$\hat{a}_{24} = 0.2439$

Labor Supply Function (3.17): $E(L|X) = .4978\ (X-2.8)$

TABLE VI-1.1973.6

Estimates of Parameters for Beta II

Occupation Code: 6

Number of Observations: 1151

$\hat{K} = 3692.58$	$\hat{c}_1 = 1.3828$
$\hat{a} = 26.1593$	$\hat{c}_2 = 38.0935$
$\hat{b} = 80.3536$	$\hat{c}_3 = 3.5158$
$\hat{a}_1 = 5.8022$	$\hat{a}_{13} = 0.0459$
$\hat{a}_2 = 0.2844$	$\hat{a}_{14} = 0.0892$
$\hat{a}_3 = 0.4962$	$\hat{a}_{15} = 6.1016$
$\hat{a}_4 = 3.7120$	$\hat{a}_{16} = 1.4500$
$\hat{a}_5 = 1.3031$	$\hat{a}_{17} = 0.2987$
$\hat{a}_6 = 0.7519$	$\hat{a}_{18} = 0.0333$
$\hat{a}_7 = 1.3095$	$\hat{a}_{19} = 0.6262$
$\hat{a}_8 = 0.1929$	$\hat{a}_{20} = 0.4283$
$\hat{a}_9 = 0.5037$	$\hat{a}_{21} = 1.0082$
$\hat{a}_{10} = 0.1883$	$\hat{a}_{22} = 0.1371$
$\hat{a}_{11} = 0.6877$	$\hat{a}_{23} = 0.2426$
$\hat{a}_{12} = 0.1994$	$\hat{a}_{24} = 0.2667$

Labor Supply Function (3.17): $E(L|X) = .5509X$

TABLE VI-1.1973.7

Estimates of Parameters for Beta II

Occupation Code: 7

Number of Observations: 1120

$\hat{K} = 2185.44$	$\hat{c}_1 = 1.1400$
$\hat{a} = 25.0005$	$\hat{c}_2 = 33.9825$
$\hat{b} = 70.8693$	$\hat{c}_3 = 3.2560$
$\hat{a}_1 = 5.5609$	$\hat{a}_{13} = 0.0631$
$\hat{a}_2 = 0.2317$	$\hat{a}_{14} = 0.0647$
$\hat{a}_3 = 0.5159$	$\hat{a}_{15} = 5.9368$
$\hat{a}_4 = 3.7279$	$\hat{a}_{16} = 1.2955$
$\hat{a}_5 = 1.2650$	$\hat{a}_{17} = 0.2750$
$\hat{a}_6 = 0.7829$	$\hat{a}_{18} = 0.0189$
$\hat{a}_7 = 1.3263$	$\hat{a}_{19} = 0.4058$
$\hat{a}_8 = 0.2191$	$\hat{a}_{20} = 0.3337$
$\hat{a}_9 = 0.4618$	$\hat{a}_{21} = 0.9042$
$\hat{a}_{10} = 0.1826$	$\hat{a}_{22} = 0.1041$
$\hat{a}_{11} = 0.6678$	$\hat{a}_{23} = 0.2078$
$\hat{a}_{12} = 0.2224$	$\hat{a}_{24} = 0.2268$

Labor Supply Function (3.17): $E(L|X) = .5362\ (X-1.86)$

TABLE VI-1.1973.8

Estimates of Parameters for Beta II

Occupation Code: 8

Number of Observations: 457

$\hat{K} = 1091.40$	$\hat{c}_1 = 0.4579$
$\hat{a} = 25.7356$	$\hat{c}_2 = 30.1809$
$\hat{b} = 66.2744$	$\hat{c}_3 = 4.5602$
$\hat{a}_1 = 6.4226$	$\hat{a}_{13} = 0.0713$
$\hat{a}_2 = 0.2413$	$\hat{a}_{14} = 0.0683$
$\hat{a}_3 = 0.5333$	$\hat{a}_{15} = 5.4682$
$\hat{a}_4 = 3.7408$	$\hat{a}_{16} = 1.4990$
$\hat{a}_5 = 1.5007$	$\hat{a}_{17} = 0.2644$
$\hat{a}_6 = 0.7356$	$\hat{a}_{18} = 0.0073$
$\hat{a}_7 = 1.2316$	$\hat{a}_{19} = 0.4150$
$\hat{a}_8 = 0.2782$	$\hat{a}_{20} = 0.2163$
$\hat{a}_9 = 0.5000$	$\hat{a}_{21} = 0.9190$
$\hat{a}_{10} = 0.2037$	$\hat{a}_{22} = 0.0987$
$\hat{a}_{11} = 0.6971$	$\hat{a}_{23} = 0.1831$
$\hat{a}_{12} = 0.2048$	$\hat{a}_{24} = 0.2353$

Labory Supply Function (3.17): $E(L|X) = .4953\ (X-2.20)$

TABLE VI-1.1973.9

Estimates of Parameters for Beta II

Occupation Code: 9

Number of Observations: 637

$\hat{K} = 2464.61$	$\hat{c}_1 = 0.9516$
$\hat{a} = 21.0365$	$\hat{c}_2 = 24.1570$
$\hat{b} = 58.3225$	$\hat{c}_3 = 4.2245$
$\hat{a}_1 = 4.8827$	$\hat{a}_{13} = 0.0322$
$\hat{a}_2 = 0.1814$	$\hat{a}_{14} = 0.0618$
$\hat{a}_3 = 0.4017$	$\hat{a}_{15} = 4.2963$
$\hat{a}_4 = 3.5335$	$\hat{a}_{16} = 1.1842$
$\hat{a}_5 = 1.0881$	$\hat{a}_{17} = 0.2528$
$\hat{a}_6 = 0.6880$	$\hat{a}_{18} = 0.0203$
$\hat{a}_7 = 0.3114$	$\hat{a}_{19} = 0.5032$
$\hat{a}_8 = 0.2548$	$\hat{a}_{20} = 0.1414$
$\hat{a}_9 = 0.3810$	$\hat{a}_{21} = 0.7070$
$\hat{a}_{10} = 0.1442$	$\hat{a}_{22} = 0.1073$
$\hat{a}_{11} = 0.6477$	$\hat{a}_{23} = 0.2442$
$\hat{a}_{12} = 0.1705$	$\hat{a}_{24} = 0.2006$

Labor Supply Function (3.17): $E(L|X) = .4796X$

TABLE VI-1.1973.11

Estimates of Parameters for Beta II

Occupation Code: 11

Number of Observations: 1363

$\hat{K} = 918.37$	$\hat{c}_1 = 1.7124$
$\hat{a} = 15.1923$	$\hat{c}_2 = 2.5440$
$\hat{b} = 40.1951$	$\hat{c}_3 = 16.7334$
$\hat{a}_1 = 3.9028$	$\hat{a}_{13} = 0.0027$
$\hat{a}_2 = 0.0883$	$\hat{a}_{14} = 0.0376$
$\hat{a}_3 = 0.2109$	$\hat{a}_{15} = 2.1325$
$\hat{a}_4 = 2.4980$	$\hat{a}_{16} = 1.5331$
$\hat{a}_5 = 1.1292$	$\hat{a}_{17} = 0.2539$
$\hat{a}_6 = 0.7962$	$\hat{a}_{18} = 0.0224$
$\hat{a}_7 = 0.5444$	$\hat{a}_{19} = 0.5447$
$\hat{a}_8 = 0.1372$	$\hat{a}_{20} = 0.0772$
$\hat{a}_9 = 0.1984$	$\hat{a}_{21} = 0.3309$
$\hat{a}_{10} = 0.0111$	$\hat{a}_{22} = 0.0989$
$\hat{a}_{11} = 0.4840$	$\hat{a}_{23} = 0.0298$
$\hat{a}_{12} = 0.0178$	$\hat{a}_{24} = 0.1103$

Labor Supply Function (3.17): $E(L|X) = .0703X$

C. Estimates for the Entire Sample

Parameter estimates for the entire sample are calculated as weighted averages of the estimates for individual occupation codes. The weights are ratios of the number of observations in the occupation code divided by the total number of observations. The weights sum to one as required. Occupation codes having singular covariance matrices were excluded from these weighted average calculations. It would be useful to include the estimates from the other cases in the weighted average calculations and this will be done when they are available.

The weighted averages of parameter estimates are calculated as follows. Let $\hat{\phi}_j^{(\nu)}$ be the j^{th} one of $\hat{a}_1, \ldots, \hat{a}_{24}, \hat{c}_1, \hat{c}_2, \hat{c}_3, \hat{b}$ for the ν^{th} occupation code. Let N_ν be the number of acceptable observations in that occupation code. Then

(6.1) $$N = \Sigma N_\nu \; .$$

Let $\hat{\phi}_j$, $j = 1, \ldots, 28$, be the weighted average estimate of the population parameters. Then

(6.2) $$\phi_j = \frac{1}{N}\Sigma\, N_\nu \phi_j^{(\nu)} \qquad j = 1, \ldots, 28.$$

Three different collections of weighted average parameter estimates have been calculated. There are two sets for 1972, one including and one not including occupation code 12. These are presented in TABLES VI-2 and VI-3. TABLE VI-4 contains parameter estimates for 1973 exclusive of occupation codes 10 and 12. Together with estimates for individual occupation codes these estimates of SPM Beta II parameters constitute a major foundation for studying consumer and worker behavior using superpopulation modeling techniques. Opportunity for analysis interpretation and extension of these results is now available. Further studies will be reported as analysis and interpretation are completed. Brief analysis of labor supply functions is contained in the next section.

TABLE VI-2

1972 Weighted Average Parameter Estimates

(Including Occupation Code 12)

$\hat{a} = 22.793$	$\hat{c}_1 = 1.335$
$\hat{b} = 62.674$	$\hat{c}_2 = 26.556$
	$\hat{c}_3 = 5.6095$
$\hat{a}_1 = 4.8579$	$\hat{a}_{13} = 0.0333$
$\hat{a}_2 = 0.2174$	$\hat{a}_{14} = 0.0811$
$\hat{a}_3 = 0.3811$	$\hat{a}_{15} = 4.6225$
$\hat{a}_4 = 3.6321$	$\hat{a}_{16} = 1.3876$
$\hat{a}_5 = 1.1771$	$\hat{a}_{17} = 0.3003$
$\hat{a}_6 = 0.8405$	$\hat{a}_{18} = 0.0282$
$\hat{a}_7 = 1.0441$	$\hat{a}_{19} = 0.7370$
$\hat{a}_8 = 0.2366$	$\hat{a}_{20} = 0.2409$
$\hat{a}_9 = 0.4902$	$\hat{a}_{21} = 0.8250$
$\hat{a}_{10} = 0.1285$	$\hat{a}_{22} = 0.1410$
$\hat{a}_{11} = 0.7224$	$\hat{a}_{23} = 0.2973$
$\hat{a}_{12} = 0.1449$	$\hat{a}_{24} = 0.2261$

TABLE VI-3

1972 Weighted Average Parameter Estimates

(Not Including Occupation Code 12)

$\hat{a} = 22.679$	$\hat{c}_1 = 1.3303$
$\hat{b} = 62.352$	$\hat{c}_2 = 26.418$
	$\hat{c}_3 = 5.6482$
$\hat{a}_1 = 4.8680$	$\hat{a}_{13} = 0.0324$
$\hat{a}_2 = 0.2152$	$\hat{a}_{14} = 0.0807$
$\hat{a}_3 = 0.3822$	$\hat{a}_{15} = 4.5791$
$\hat{a}_4 = 3.5902$	$\hat{a}_{16} = 1.4015$
$\hat{a}_5 = 1.1791$	$\hat{a}_{17} = 0.2998$
$\hat{a}_6 = 0.8344$	$\hat{a}_{18} = 0.0284$
$\hat{a}_7 = 1.0341$	$\hat{a}_{19} = 0.7338$
$\hat{a}_8 = 0\ 2341$	$\hat{a}_{20} = 0.2300$
$\hat{a}_9 = 0.4873$	$\hat{a}_{21} = 0.8151$
$\hat{a}_{10} = 0.1277$	$\hat{a}_{22} = 0.1392$
$\hat{a}_{11} = 0.7187$	$\hat{a}_{23} = 0.2998$
$\hat{a}_{12} = 0.1437$	$\hat{a}_{24} = 0.2243$

TABLE VI-4

1973 Weighted Average Parameter Estimates

(Not Including Occupation Code 12)

$\hat{a} = 25.604$	$\hat{c}_1 = 1.9261$
$\hat{b} = 70.7$	$\hat{c}_2 = 30.324$
	$\hat{c}_3 = 6.7236$
$\hat{a}_1 = 5.5234$	$\hat{a}_{13} = 0.0471$
$\hat{a}_2 = 0.2451$	$\hat{a}_{14} = 0.0841$
$\hat{a}_3 = 0.3946$	$\hat{a}_{15} = 5.1472$
$\hat{a}_4 = 4.1144$	$\hat{a}_{16} = 1.5431$
$\hat{a}_5 = 1.3057$	$\hat{a}_{17} = 0.3132$
$\hat{a}_6 = 0.9406$	$\hat{a}_{18} = 0.0307$
$\hat{a}_7 = 1.2666$	$\hat{a}_{19} = 0.7642$
$\hat{a}_8 = 0.2353$	$\hat{a}_{20} = 0.3044$
$\hat{a}_9 = 0.5270$	$\hat{a}_{21} = 0.9549$
$\hat{a}_{10} = 0.1510$	$\hat{a}_{22} = 0.1549$
$\hat{a}_{11} = 0.8059$	$\hat{a}_{23} = 0.3315$
$\hat{a}_{12} = 0.1768$	$\hat{a}_{24} = 0.2425$

D. Per Capita Labor Supply Estimates

Per capita labor supply functions are given by equation (3.17) in chapter III. The corresponding estimated function is shown at the bottom of each table of estimates. These labor supply function estimates indicate the alternative characteristics of different occupation code populations.

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